February 24, 1887.

Professor STOKES, D.C.L., President, in the Chair.

The Presents received were laid on the table, and thanks ordered for them.

The following Papers were read:—

I. "Problems in Mechanism regarding Trains of Pulleys and Drums of Least Weight for a given Velocity Ratio." By HENRY HENNESSY, F.R.S., Professor of Applied Mathematics and Mechanism in the Royal College of Science, Dublin. Received February 7, 1887.

Eighty years have elapsed since Dr. Thomas Young* published a theorem which has since found a place in most of the scientific treatises on mechanism. This theorem states that in order to obtain a given value or velocity ratio by a train of toothed wheels and pinions of which all the pairs are equal, the ratio of the number of teeth in each wheel to the number in each pinion should be as 359 to 100, when the total number of teeth in the train is the least possible. The late Professor Willis has remarked that the rule deduced from this theorem seemed not to have much practical utility, but he illustrates his remarks by referring to the trains of wheels and pinions employed in clockwork. As trains of wheels, pulleys, and drums, are largely employed in many machines whose arrangements greatly differ from clockwork, and especially in the processes of textile manufacture, it may be interesting to examine whether there are not other conditions, besides the number of teeth, which may be economised in the transformation of a movement of rotation from a moderate rate of velocity to a very high rate of velocity.† As the number of teeth on a wheel or pinion is proportional to the circumference of the pitch-

* 'Natural Philosophy,' vol. 2, p. 56. 4to. 1807. The preface to this volume is dated March, 1807.

† In some spinning machines it is said that the spindles rotate with velocities of from 6000 to 7000 turns per minute, and high velocities are also often required for reels, bobbins, and fliers. Between these rapidly rotating parts of the machines and the prime mover, trains of pulleys, drums, or wheels are usually interposed, the value of each such train depending on the required increase of velocity.—[Feb. 21, 1887.]

circle, it may be understood as giving a rule for deducing the ratio of the diameters of the wheels and pinions so that the sum of all their circumferences shall be a minimum. Although economy of the circumferences of wheels, speed pulleys or drums in a train may not be of much importance, is it not possible that economy of total weight of material employed may be worthy of inquiry? Reduction of weight in the parts of a machine is not merely economy of materials employed in the structure, but in the case of moving parts it involves economy of work by lessening the resistances due to friction. The following problems have arisen from such considerations, and in all of them, as well as in that studied by Young, if we call m the number of similar pairs of wheels, speed pulleys or drums, C the circumference of a large wheel, &c., and c of a small one in the same train, the velocity ratio or value of the train u will be—

$$u = (\mathrm{C}/c)^m = (\mathrm{R}/r)^m = x^m,$$

where x represents the ratio of the radii R and r of a large and a small wheel or pulley. In all such problems we have therefore

$$m = \log u/\log x,$$

and whether the question relates to the volume or circumference of the wheels or pulleys the usual operations of the calculus will in every case lead to a minimum.

The volumes or circumferences of pairs of pulleys or wheels with radii having the ratio x may in general be expressed in the form $\mathrm{F}x = a+bx+cx^2$, where a, b, and c are constants. On multiplying this by m we have—

$$\mathrm{V} = \frac{\log u}{\log x}\mathrm{F}x.$$

Hence
$$\frac{1}{\log u}\frac{d\mathrm{V}}{dx} = \frac{\mathrm{F}'x}{\log x} - \frac{\mathrm{F}x}{x(\log x)^2},$$

$$\frac{1}{\log u}\frac{d^2\mathrm{V}}{dx^2} = \frac{\mathrm{F}''x}{\log x} - \frac{2\mathrm{F}'x}{x(\log x)^2} + \frac{\mathrm{F}x}{x^2(\log x)^2} + \frac{2\mathrm{F}x}{x^2(\log x)^3}$$

$$= \frac{\mathrm{F}''x}{\log x} + \frac{\mathrm{F}x}{x^2(\log x)^2} - \frac{2}{x\log x}\left(\frac{\mathrm{F}'x}{\log x} - \frac{\mathrm{F}x}{x(\log x)^2}\right).$$

But as
$$\frac{d\mathrm{V}}{dx} = 0, \qquad \frac{\mathrm{F}'x}{\log x} - \frac{\mathrm{F}x}{x(\log x)^2} = 0,$$

and
$$\frac{d^2\mathrm{V}}{dx^2} = \frac{\log u}{\log x}\left(\mathrm{F}x'' + \frac{\mathrm{F}x}{x^2\log x}\right),$$

and from the form of $F'x$ in these problems $F''x$ and Fx are positive, therefore d^2V/dx^2 must be always a positive quantity; whence the value of x obtained in all such problems makes V a minimum.

Small pulleys carrying cords are usually made solid and approximately cylindrical; in a train of such pulleys the volumes of the large and small cylinders may be denoted by $\pi b R^2$ and $\pi b r^2$, where b is the common thickness of each cylindrical disk; the total volume of the train will therefore be—

$$V = m\pi b(R^2+r^2) = m\pi b r^2(R^2/r^2+1),$$

or using the preceding notation,

$$V = \frac{\pi b r^2 \log u}{\log x}(1+x^2) = K\frac{(1+x^2)}{\log x},$$

$$\frac{1}{K}\frac{dV}{dx} = \frac{2x^2\log x-(1+x^2)}{x(\log x)^2},$$

which gives $$\log x = \tfrac{1}{2}(1+x^{-2});$$

this equation is approximately satisfied by making $x = 1{\cdot}895$. Hence the ratio of the radii may be practically set down as 19 to 10 for a train of pulleys of minimum volume or least weight of material.

In drums the surface carrying the band is broad, and this surface is commonly supported by spokes which radiate from the axle, while sometimes, as in pulleys, the drum consists of a disk with a broad hoop. If the thickness of the hoop and its disk are equal, a problem similar to the foregoing can be easily solved. The question is, in a series of large and small drums if all the large are equal and also all the small, required the ratio of their diameters so that the entire train shall have the least volume for a given velocity ratio. Let t be the uniform thickness of the disks and hoops of the drums, R and r the radii of a small and a large disk, b the breadth of the hoops; we shall have for the total volume of the train

$$V = m\pi[2(R+r)tb+t(R^2+r^2)],$$

when t is so small compared to R, r, and b, that quantities multiplied by t^2, &c., may be omitted.

The above may be written

$$V = \frac{\pi r^2 t\log u}{\log x}[2(x+1)b/r+x^2+1],$$

which gives, by the usual process of making $dV/dx = 0$,

$$\log x = \frac{2(x+1)b/r+x^2+1}{2x(b/r+x)}.$$

In the particular case where r is a multiple of b, or $r = nb$,

$$\log x = \frac{2(x+1)+n(x^2+1)}{2x(nx+1)};$$

and if $n = 1$,

$$\log x = \frac{1+(x+1)^2}{2x(x+1)}.$$

This equation gives $x = 2{\cdot}21$ nearly, or practically a ratio of 11 to 5 for the diameters of the large and small drums in such a train as has been indicated.

Although it is manifest that the volume of a single pair of pulleys with the same velocity ratio as this train of five pairs would be considerably greater, it may be interesting to make the comparison. If R′ be the radius of the large pulley in the single pair, and as before r of the small pulley, then $R' = ur$, and the volume of the pair $V' = \pi t(u^2+1)r^2$. As before, the volume of the train is $V = \pi nt(x^2+1)r^2$.

Hence $$\frac{V'}{V} = \frac{u^2+1}{n(x^2+1)} = \frac{x^{2n}+1}{n(x^2+1)}.$$

If $x = 1{\cdot}9$ and $n = 5$, we shall have $V'/V = 26{\cdot}64$, or the volume of a single pair would be more than twenty-six times the volume of a train of five pairs with the same velocity ratio.

Another solution can be easily found if a train of drums were so constructed that the volume of the spokes supporting the hoop of each drum would be half the volume of a complete disk, in this case

$$V = m\pi t\,[2(R+r)b+\tfrac{1}{2}(R^2+r^2)]$$

$$= \pi r^2 t\,[2(x+1)\frac{b}{r}+\tfrac{1}{2}(x^2+1)]\frac{\log u}{\log x},$$

and if we make $b = r$, this gives, from $dV/dx = 0$,

$$\log x = \frac{(x+2)^2+1}{2x(x+2)},$$

which is satisfied by making $x = 2{\cdot}55$, or the diameters of the large drums would be to those of the small drums in the ratio of 51 to 20 in a train of least weight of drums such as here described.

If in this case b in all the drums instead of being equal to r was equal to the greater radius R, we would have evidently

$$V = m\pi r^2 t[2(x+1)x+\tfrac{1}{2}(x^2+1)]$$

$$= \tfrac{1}{2}\pi r^2 t[5x^2+4x+1]\,\frac{\log u}{\log x},$$

and when $dV/dx = 0$,

$$\log x = \frac{5x^2+4x+1}{10x^2+4x}.$$

This will be satisfied by making x somewhat less than 1·9, so that in this case the ratio of the diameters of the drums would be a little less and very close to the ratio found for the pulleys.

In order to illustrate the foregoing problems a model of a train of pulleys and another of a train of drums made of brass were constructed by Mr. Yates. In the train of pulleys all the large ones are 1·9 inches in diameter, and all the small are 1 inch. Each of the former weighs 2·61 oz., and each of the latter 1·058 oz.; as there are five pairs their total weight is 18·340 oz., while they give a velocity ratio of $(1{\cdot}9)^5 = 24{\cdot}761$, or a little more than $24\frac{3}{4}$.

The train of drums consists of large ones with diameters of 2·55 inches and small of 1 inch, the hoops are in all 0·5 inch in breadth, and the spokes are half the volume of a complete disk. The weights of the large drums are each 3·386 oz., of the small 0·811 oz.

There are four pairs of drums, and their total weight is 16·788 oz., or little more than 1 lb.

The velocity ratio of this train is $(51/20)^4 = 42{\cdot}2825$, or a little more than $42\frac{1}{4}$.

II. "On the Relation between Tropical and Extra-Tropical Cyclones." By Hon. RALPH ABERCROMBY, F.R. Met. Soc. Communicated by R. H. SCOTT, M.A., F.R.S. Received February 7, 1887.

(Abstract.)

The conclusions as to the relation of tropical to extra-tropical cyclones which the author has derived from the researches of which this paper gives an account, may be stated thus:—

All cyclones have a tendency to assume an oval form; the longer diameter may lie in any direction, but has a decided tendency to range itself nearly in a line with the direction of propagation.

The centre of the cyclone is almost invariably pressed toward one or other end of the longer diameter, but the displacement may vary during the course of the same depression.

Tropical hurricanes are usually of much smaller dimensions than extra-tropical cyclones; but the central depression is much steeper, and more pronounced in the former than in the latter.

Tropical cyclones have less tendency to split into two, or to develope secondaries, than those in higher latitudes.

A typhoon which has come from the tropics can combine with a